MSC INTERNAL NOTE NO. 64-EA-29

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A THEORETICAL METHOD FOR PREDICTING PURE ROTATIONAL AERODYNAMIC DAMPING DERIVATIVES FROM STATIC FORCE AND MOMENT COEFFICIENTS

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
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JUNE 1, 1964

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A THEORETICAL METHOD FOR PREDICTING PURE ROTATIONAL AERODYNAMIC DAMPING DERIVATIVES FROM STATIC FORCE AND MOMENT COEFFICIENTS

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SYMBOLS

θ, Angle between V_∞ and vehicle centerline.

$\dot{\theta}$, $\frac{d\theta}{dt}$

$\ddot{\theta}$, $\frac{d^2\theta}{d_t{}^2}$

$F(\theta)$, General damping function

$G(\theta)$, General Forcing Function

α, Angle of Attack

q_∞, Free stream Dynamic Pressure

q, Instantaneous dynamic pressure

V_∞, Free stream velocity

V_N, Total velocity normal to body centerline

V_A, Total velocity parallel to body centerline

V_T, Total velocity

ρ, Density

S, Reference length

I, Mass moment of inertia about center of gravity

C_N, Normal force coefficient

C_A, Axial force coefficient

C_L, Lift force coefficient

C_D, Drag force coefficient

C_M, Pitching moment coefficient about center of gravity

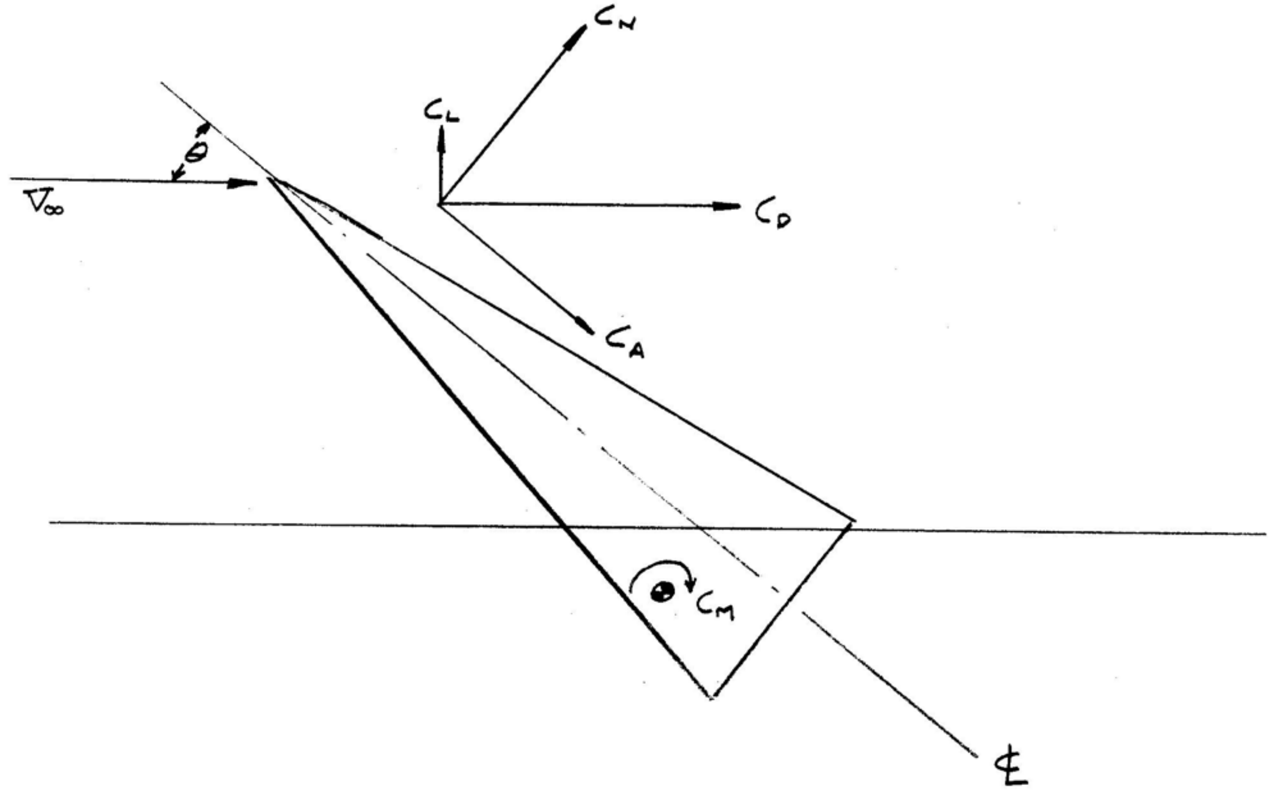
C_{M_α}, $\frac{CM}{d\alpha}$

C_R, Total force coefficient

r, CM/CR

t, Time

Sign Convention for force coefficients

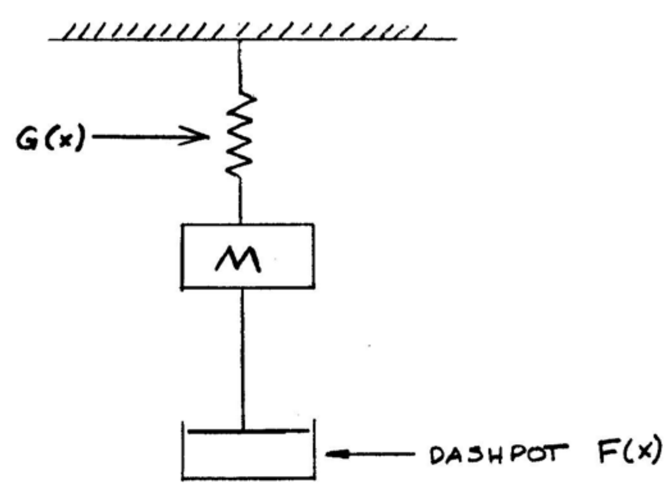


Introduction:

In order to investigate the angular motion of an aerodynamic vehicle; consideration should be given to the aerodynamic damping caused by the vehicle's rotational rate. When vehicle orientation is important to the success of a mission, as in the case of many abort configurations, the aerodynamic damping can be of particular interest. With this need to determine the the dynamic stability of many configurations, it is evident that a theoretical method for calculating the aerodynamic damping derivatives, $C_{M_q} + C_{M_{\dot{\alpha}}}$, would be of significant value. A technique for predicting the damping characteristics of a vehicle from its static force and moment data has been developed by the Analytical Aerodynamics Section of the Advanced Spacecraft Technology Division, Aerodynamics Branch. The equation of motion for an oscillating vehicle was rewritten to include the effects that pitch rates have on the local dynamic pressure and angle-of-attack. Utilizing this equation a computer program was able to predict the experimental results of a particular vehicle. The development, application, and results of this study will be presented.

Analysis:

An analogy of the single-degree-of-freedom of an aerodynamic vehicle can be seen in the motion of a weight on a spring with restriant by a dashpot.



In this example G (x) is a forcing function, while F(x) is a damping function. Displacement, x, of the weight is dependent on both G (x) and F (x). The differential equation describing the motion of the weight is:

$$(1) \quad M\ddot{x} = G(x) + F(x)\dot{x}$$

For the case of an aerodynamic vehicle, the differential equation for rotational motion is of the same form, with the substitution of corresponding angular quantities for translational quantities.

That is:

$$(2) \quad I\ddot{\theta} = G(\theta) + F(\theta)\dot{\theta}$$

Here G (θ) is the vehicle's pitching moment, and F (θ) is a function of the aerodynamic damping derivatives, $C_{m_q} + C_{m_{\dot{\alpha}}}$. Assuming

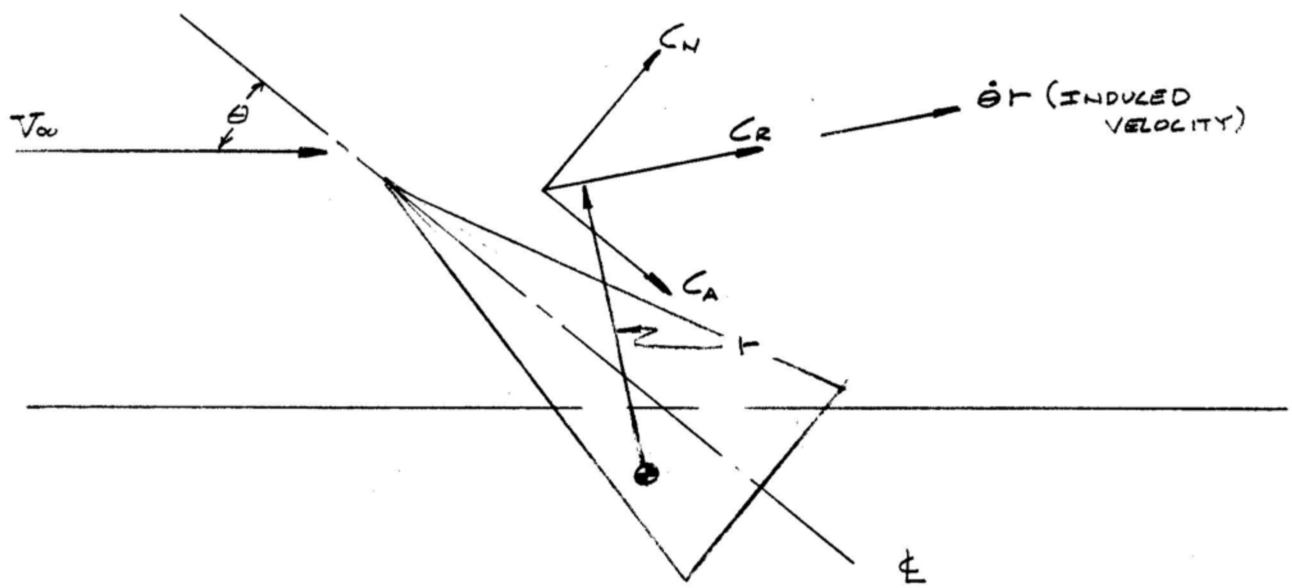
$$(3) \quad F(\theta) = (C_{m_q} + C_{m_{\dot{\alpha}}}) \frac{q_\infty S D^2}{2 V_\infty}$$

In general, forcing and damping functions are non-linear functions of θ. The damping function is a dominant factor in the dynamic stability characteristics of the vehicle. Depending on the sign of F (θ) the vehicle can damp out its oscillatory motion (dynamically stable) or diverge to the point of tumbling (dynamically unstable).

Up to the present, mathematical approaches to equation (2) have proven futile. In many of the theoretical approaches, problems can be encountered with CN is zero and/or CA does not act along the centerline of the vehicle. The method discussed in this paper circumvents these problems by introducting a resultant force coefficient, C_R The technique is applicable to non-symmetrical and symmetrical and symmetrical off-set center of gravity type vehicles. Attention will be on the damping caused by change in angle of attack and dynamic pressure due to angular pitch velocity.

In order to account for the damping of the total vehicle, it is necesary to picture the configuraton as the sum of its differential elements. Since obtaining aerodynamic coefficients as continuous functions of body length is difficult, this approach proves unrealistic. Dividing the body into the major components that contribute to the total aerodynamics is a reasonable compromise. Because the damping due to each of these components is calculated in exactly the same manner, the following derivation will be based upon determining the damping for one of these components. The total damping of the vehicle is then the sum of the damping contributions of each individual component.

Damping due to one compoment:



First, some attention will be paid to the total force coefficient.

$C_R = \sqrt{C_N^2 + C_A^2} = \sqrt{C_L^2 + C_D^2}$ and represents the total force on the component.

Knowing that the pitching moment is caused by a single force times some lever arm, the C_M can be defined as follows:

$$C_M = \left(\frac{\vdash}{D}\right) C_R$$

$$\vdash/D = C_M / C_R$$

The intersection of the lever arm, $\vdash$ and the total force's line of action, is the location of resultant aerodynamics on the component and is the logical point to examine the changes in angle-of-attack and dynamic pressure due to pitch velocity.

The damping derivatives are defined as follows:

$$C_{M_q} + C_{M_{\dot\alpha}} = \frac{\partial C_M}{\partial \frac{\dot\theta D}{2V_\infty}}$$

$$= \frac{\partial M}{\partial \dot\theta} \times \frac{2V_\infty}{D} \times \frac{1}{q_\infty S D}$$

$$= \frac{\partial\, C_M\, q S D}{\partial \dot\theta} \times \frac{2V_\infty}{q_\infty S D^2}$$

Since the only quantities in the above equation that change with angular velocity are C_M AND q:

$$C_{M_q} + C_{M_{\dot\alpha}} = \frac{\partial\, C_M\, q}{\partial \dot\theta} \times \frac{2V_\infty}{q_\infty D}$$

Here note is made of the fact that the pitching moment coefficient is an implicit function of $\dot\theta$, so that:

$$C_{M_q} + C_{M_{\dot\alpha}} = \frac{2V_\infty}{q_\infty D}\left[q \frac{\partial C_M}{\partial \alpha} \cdot \frac{\partial \alpha}{\partial \dot\theta} + C_M \frac{\partial q}{\partial \dot\theta}\right]$$

$$= \frac{2V_\infty}{q_\infty D}\left[q\, C_{M_\alpha} \frac{\partial \alpha}{\partial \dot\theta} + C_M \frac{\partial q}{\partial \dot\theta}\right]$$

Now the changes in angle of attack and dynamic pressure with respect to $\dot{\theta}$ must be established. Observing that the rotation of $\dot{\theta}$ causes and induced velocity, $\dot{\theta} r$ (See figure 2), the angle-of-attack and dynamic pressure can be defined as functions of $\dot{\theta}$.

A. $\dfrac{\partial \alpha}{\partial \dot{\theta}}$

The angle of attack can be defined as the arctan of the ratio of net velocity in the normal direction to the net velocity in the axial direction.

$$\alpha = \tan^{-1} \frac{V_N}{V_A}$$

Since the resultant induced velocity vector, $\dot{\theta} r$, lies along the same line of action as the total force:

$$V_N = V_\infty \sin\theta - \dot{\theta} r \, C_N/C_R$$

$$V_A = V_\infty \cos\theta - \dot{\theta} r \, C_A/C_R$$

$$\alpha = \tan^{-1} \frac{V_\infty \sin\theta - \dot{\theta} r \, C_N/C_R}{V_\infty \cos\theta - \dot{\theta} r \, C_A/C_R}$$

$$\frac{\partial \alpha}{\partial \dot{\theta}} = \frac{\partial}{\partial \dot{\theta}} \left[\tan^{-1} \frac{V_\infty \sin\theta - \dot{\theta} r \, C_N/C_R}{V_\infty \cos\theta - \dot{\theta} r \, C_A/C_R} \right]$$

$$= \frac{\left(V_\infty \cos\theta - \dot{\theta} r \, C_A/C_R\right)\left(-r \, C_N/C_R\right) - \left(V_\infty \sin\theta - \dot{\theta} r \, C_N/C_R\right)\left(-r \, C_A/C_R\right)}{\left[1 + \left(\dfrac{V_\infty \sin\theta - \dot{\theta} r \, C_N/C_R}{V_\infty \cos\theta - \dot{\theta} r \, C_A/C_R}\right)^2\right] \left(V_\infty \cos\theta - \dot{\theta} r \, C_A/C_R\right)^2}$$

$$= \frac{-V_\infty r \, C_N/C_R \cos\theta + \dot{\theta} r^2 \, C_A C_N/C_R + V_\infty r \, C_A/C_R \sin\theta - \dot{\theta} r^2 \, C_N C_A/C_R}{\left(V_\infty \cos\theta - \dot{\theta} r \, C_A/C_R\right)^2 + \left(V_\infty \sin\theta - \dot{\theta} r \, C_N/C_R\right)^2}$$

$$\frac{\partial \alpha}{\partial \dot{\theta}} = \frac{V_\infty \; r/C_R \left(-C_N \cos\theta + C_A \sin\theta\right)}{\left(V_\infty \cos\theta - \dot{\theta} r \; C_A/C_R\right)^2 + \left(V_\infty \sin\theta - \dot{\theta} r \; C_N/C_R\right)^2}$$

Noticing that

$C_L = C_N \cos\theta - C_A \sin\theta$ and that the denominator is $V_A^2 + V_N^2 = V_T^2$, the above equation becomes:

$$\frac{\partial \alpha}{\partial \dot{\theta}} = \frac{-V_\infty \; C_L/C_R \; r}{V_T^2}$$

B. $\frac{\partial q}{\partial \dot{\theta}}$

$$q = \tfrac{1}{2}\rho V_T^2 = \tfrac{1}{2}\rho\left[\left(V_\infty \cos\theta - \dot{\theta} r \; C_A/C_R\right)^2 + \left(V_\infty \sin\theta - \dot{\theta} r - C_N/C_R\right)^2\right]$$

$$\frac{\partial q}{\partial \dot{\theta}} = \tfrac{1}{2}\rho\left[2\left(V_\infty \cos\theta - \dot{\theta} r \; C_A/C_R\right)\left(-r \; C_A/C_R\right) + 2\left(V_\infty \sin\theta - \dot{\theta} r \; C_N/C_R\right)\left(-r \; C_N/C_R\right)\right]$$

$$= \rho\left[-V_\infty r \; C_A/C_R \cos\theta + \dot{\theta} r^2 \; C_A^2/C_R^2 - V_\infty r \; C_N/C_R \sin\theta + \dot{\theta} r^2 \; C_N^2/C_R^2\right]$$

$$= \rho\left[-V_\infty \frac{r}{C_R}\left(C_A \cos\theta + C_N \sin\theta\right) + \dot{\theta} r^2 \; \frac{C_A^2 + C_N^2}{C_R^2}\right]$$

$C_D = C_A \cos\theta + C_N \sin\theta$ $\qquad\qquad$ $C_R^2 = C_A^2 + C_N^2$

$$\frac{\partial q}{\partial \dot{\theta}} = \rho\left(-V_\infty \frac{C_D}{C_R} r + \dot{\theta} r^2\right)$$

Substitution of equations 5, 6, and 7 into equation 4 yields:

$$C_{m_q} + C_{m_{\dot\alpha}} = \frac{2V_\infty}{q_\infty D}\left[\frac{1}{2}\rho V_T^2 C_{m_\alpha}\left(\frac{-V_\infty \, C_L/C_R \, r}{V_T^2}\right) + C_m \rho\left(-V_\infty \frac{C_D}{C_R} r + \dot\theta r^2\right)\right]$$

$$= \frac{2V_\infty}{q_\infty D}\left[-\frac{1}{2}\rho C_{m_\alpha} V_\infty \frac{C_L}{C_R} r - C_m \rho V_\infty \frac{C_D}{C_R} r + C_m \rho \dot\theta r^2\right]$$

$$= -2 C_{m_\alpha} \frac{C_L}{C_R} \frac{r}{D} - 4 C_m \frac{C_D}{C_R} \frac{r}{D} + 4 \frac{C_m \dot\theta r^2}{D V_\infty}$$

Since the last term is usually small, it is reasonably safe to neglect it. Then:

$$C_{m_q} + C_{m_{\dot\alpha}} = -2 C_{m_\alpha} \frac{C_L}{C_R} \frac{r}{D} - 4 C_m \frac{C_D}{C_R} \frac{r}{D}$$

$$= \frac{C_m}{C_N^2 + C_A^2}\left[-2 C_{m_\alpha} C_L - 4 C_m C_D\right]$$

Since this is the $C_{m_q} + C_{m_{\dot\alpha}}$ for one component of the vehicle, the total damping is:

$$C_{m_q} + C_{m_{\dot\alpha}} = \sum \frac{C_m}{C_N^2 + C_A^2}\left[-2 C_{m_\alpha} C_L - 4 C_m C_D\right]$$

The more components that the vehicle is divided into, the more accurate the damping values obtained. For the ideal case of continuous aerodynamics force coefficients as a function of body length, the intergal equations:

$$C_{m_q} + C_{m_{\dot{\alpha}}} = -2\int d\left[\frac{C_m C_{m_\alpha} C_L}{C_N^2 + C_A^2}\right] - 4\int d\left[\frac{C_m^2 C_D}{C_N^2 + C_A^2}\right]$$

can be applied. But then more detailed static force data are needed. A compromise has to be reached as to the reasonable number of components. An example of component division is the Apollo Launch Escape Vehicle (LEV) with canards. In the case of the Apollo LEV with canards, the vehicle was divided into three components: Command Module, escape rocket and tower, and canards. From the static data of these components the damping was obtained by the following equation:

$$\left(C_{m_q} + C_{m_{\dot{\alpha}}}\right)_{\text{TOTAL}} = \left(C_{m_q} + C_{m_{\dot{\alpha}}}\right)_{\substack{\text{COMMAND}\\ \text{MODULE}}} + \left(C_{m_q} + C_{m_{\dot{\alpha}}}\right)_{\substack{\text{ROCKET}\\ \text{\& TOWER}}} + \left(C_{m_q} + C_{m_{\dot{\alpha}}}\right)_{\text{CANARDS}}$$

Figure 4 shows the damping derivatives calculated from the above summation equation. As a comparison, the $C_{m_q} + C_{m_{\dot{\alpha}}}$ obtained by considering the body as one component is shown in figure 4. As can be seen, the damping of the LEV canards taken as whole is less than that obtained from the component approach. Figure 3 shows the pitching moment coefficient for the total LEV with canards. It should be noted that the experimental static data covered the range from -50 to + 130 degrees angle-of-attack. The data for the remainder of the angle-of-attack range was faired in, including the trim region. In addition, the data were obtained by use of two different scale models: A 1/10-scale model tested in the Ames Unitary Plan Wind Tunnel and a 1/20-scale modle tested in the Jet Propulsion Laboratory's (JPL) 20 inch supersonic wind tunnel. These figures must be considered in evaluating the results of the theory.

In figure 5 is seen the actual angle-of-attack time history of the LEV canards obtained on the JPL AFD-1 dynamic stability test and the angle-of-attack time history calculated using the C_M and $C_{M_q} + C_{M_{\dot{\alpha}}}$ values shown in figures 3 and 4. For the larger oscillation amplitues there is close agreement, but as the amplitude decreases the two curves show less agreement. This is in the region where experimental data were not available.

In studying other configurations, it was seen that this theory has the capability of predicting both stable and unstable damping.

Further investigation of the theory is continuing in the Analytical Aerodynamics Section.

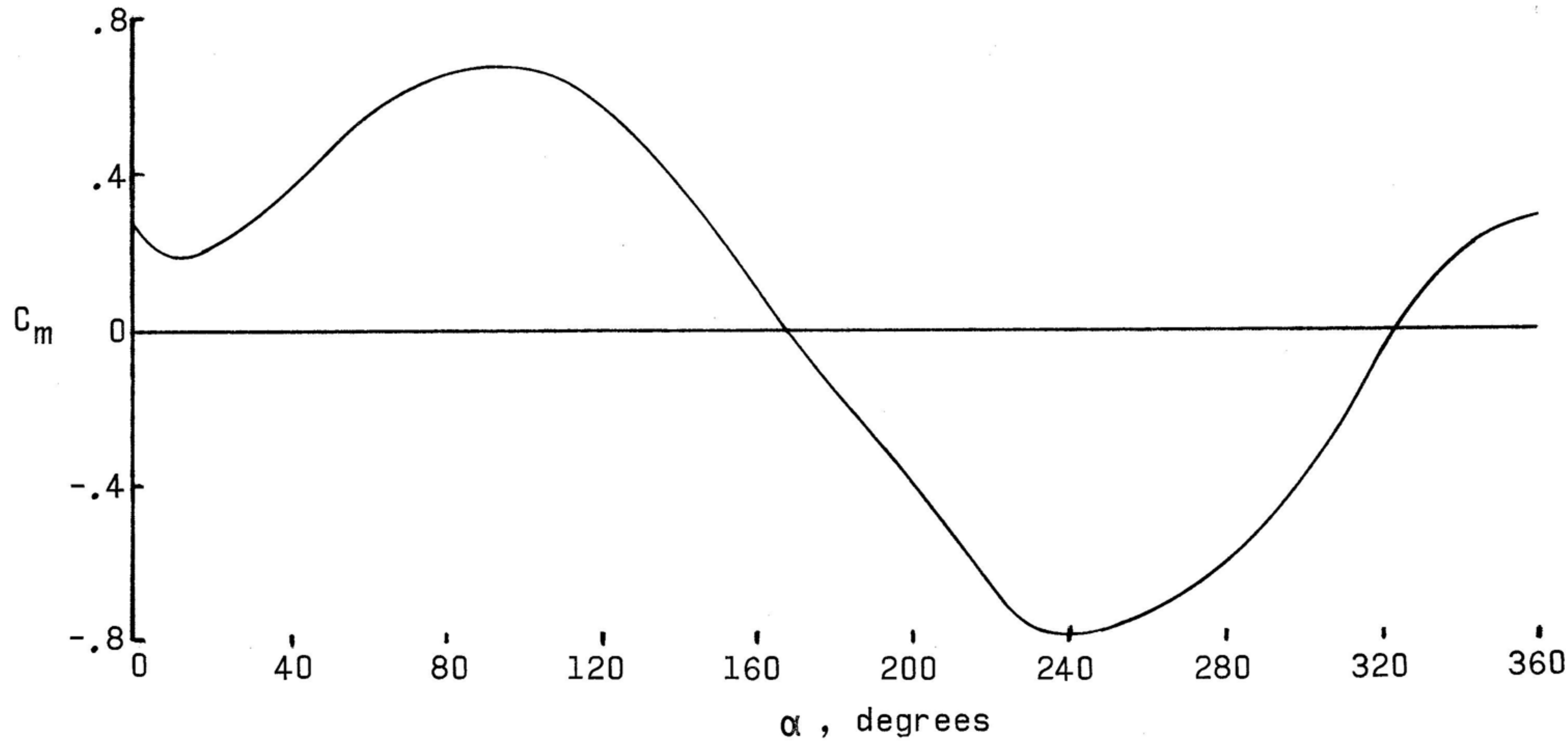


Figure 3

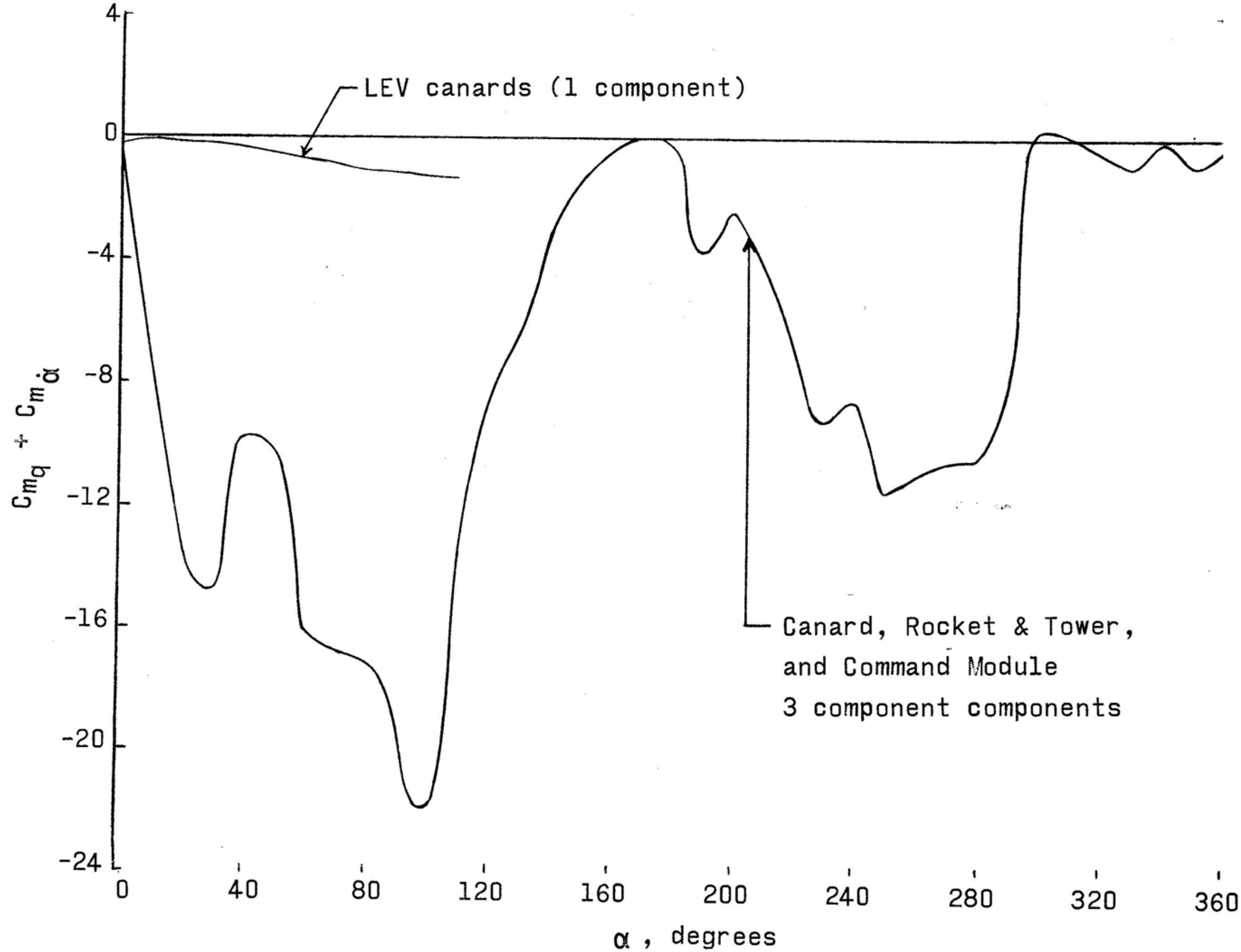
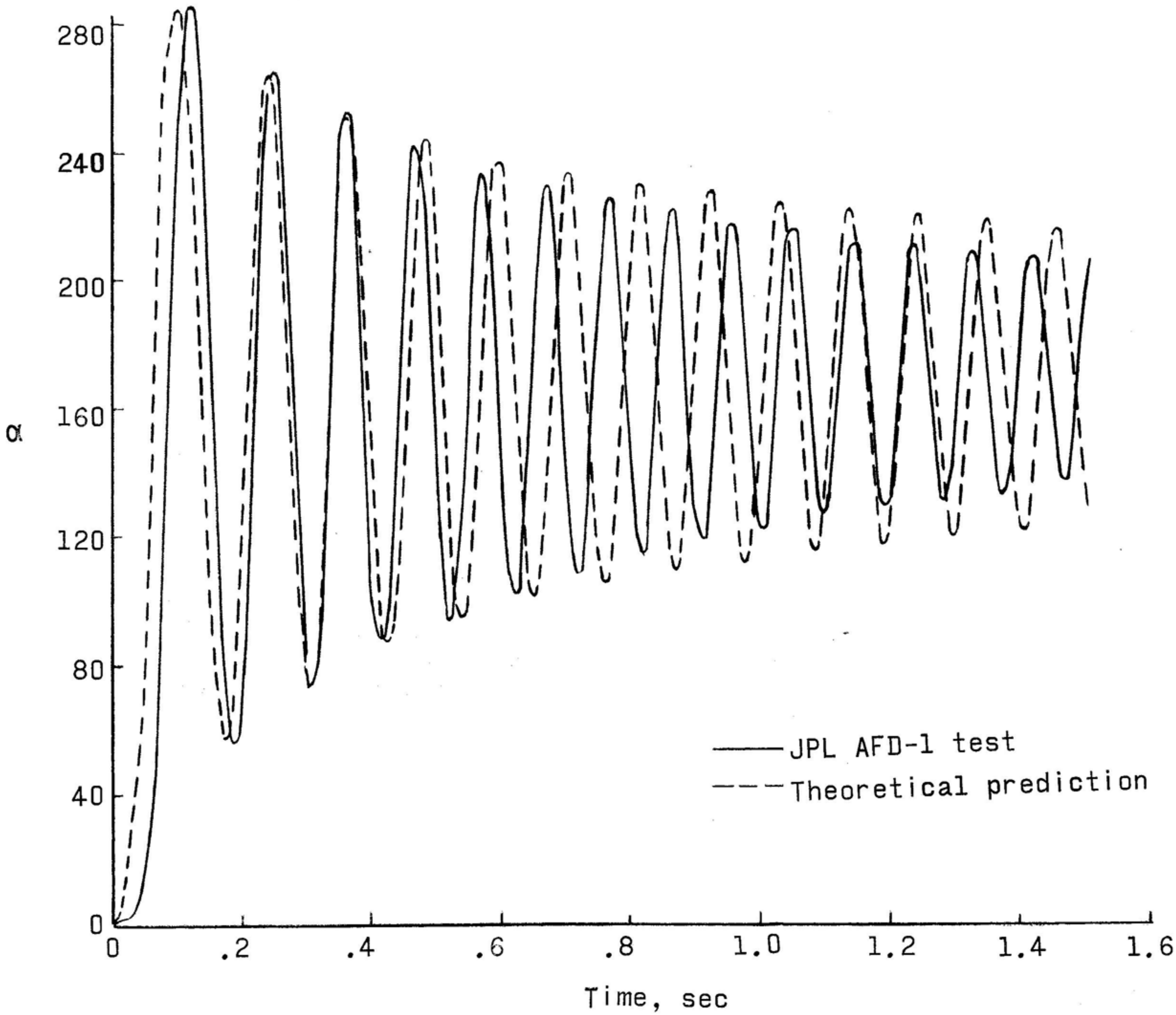

LEV canards (1 component)
Canard, Rocket & Tower,
and Command Module
3 component components
4
0
-4
-8
-12
-16
-20
-24
$C_{m_q} + C_{m_{\dot{\alpha}}}$
0
40
80
120
160
200
240
280
320
360
α , degrees

Figure 4



280
240
200
160
120
80
40
0
α
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
Time, sec
JPL AFD-1 test
Theoretical prediction

Figure 5